A model of inequality aversion and public goods when the marginal rate of substitution is not constant

Hide-Fumi Yokoo
National Institute for Environmental Studies, Japan
16-2 Onogawa, Tsukuba, Ibaraki 305-8506, Japan
E-mail: yokoo.hidefumi@nies.go.jp

May 9, 2018

Abstract

This paper develops a model of inequality aversion and public goods, which allows the case of the marginal rate of substitution being variable. The utility function of the standard public goods model is nested in the Fehr-Schmidt model. As such, an individual's contribution function for the public good is derived by solving the problem of a utility function that has a kink, and examining both interior and corner solutions. Due to the variable marginal rate of substitution and the preferences kinkiness, the derived contribution function is not monotonic with respect to the other's provision.

Keywords: Inequality Aversion, Kinky Preferences, Private Provision of Public Goods, Variable Marginal Rate of Substitution

JEL Codes: H41, D63, D91

1. Introduction

Several studies examine the association between inequality aversion and the voluntary provision of public goods (Buckley and Croson, 2006; Blanco et al., 2011; Teyssier, 2012) by testing the models of inequality aversion, including the one developed by Fehr and Schmidt (1999), through laboratory experiments. A linear public goods game is used, by assuming the marginal rate of substitution (MRS) between the private and public good constant. Although, the original Fehr-Schmidt model suffices to explain decision making in a standard linear public goods game, the assumption made by previous studies (the constant MRS) may be too strong for the demand for public goods outiside the laboratory.

This paper thus extends the model of inequality aversion and public goods to incorporate the case when the MRS is not constant. To this end, a model of a two-person economy is developed to study individuals' contribution to a public good. The theoretical foundation of the model is provided by nesting the utitlity function of the standard public goods model (Warr, 1983; Bergstrom et al., 1986) into the model by Fehr and Schmidt (1999). Further, a maximization problem with a utility function with a kink is presented. To obtain the intuition of the model, a Cobb-Douglas function is adopted, and the problem is solved to derive an optimal response. A clear and empirically testable prediction is derived by the comparative statics.

Engelmann (2012) shows that extending the Fehr-Schmidt model by adding a term for efficiency concerns is misguided, since it is equivalent to a much simpler change. This argument also applies to adding a term for public goods if MRS is constant. Instead of an attempt such as those criticized by Engelmann (2012), this study adds a term that is a concave function of the total amount of the public good provision.

Derin-Güre and Uler's (2010) study is an exception in taking a novel approach to model inequality aversion and public goods. Their utility function includes, in addition to standard private and public good terms, a term of the concave function of inequality in private good consumption. Their utility function exhibits a variable MRS that is assumed to be differentiable. Consequently, an interior solution is focused.

Unlike Derin-Güre and Uler (2010), this paper proposes a utility function that is non-differentiable by nature and examines both interior and corner solutions. Further, a simple algorithm to solve a problem with a non-differentiable function is provided to derive the optimal contribution as a function of other's contribution. The derived optimal contribution exhibits an inverted N-shape relationship, which means that an individual decreases its contribution in response to the other's increase over a specific range, but increases it in the other ranges.

2. Model

Consider an economy consisting two individuals, individual A and B. Assume that there are only a private and a public good. Each individual i consumes an amount x_i of the private good and donates an amount g_i to the supply of the public good. Let $G = g_A + g_B$ be the total private contributions to the public good. Both individuals i are endowed with wealth $w > 0$, which they allocate between private good x_i and contribution g_i. Let $\pi_i = \pi(x_i, G)$ be individual i's sub-utility, which corresponds to monetary payoffs in the model of Fehr and Schmidt (1999). Assume that $\frac{\partial \pi}{\partial x} \geq 0$, $\frac{\partial^2 \pi}{\partial x^2} \leq 0$, $\frac{\partial \pi}{\partial G} \geq 0$, and $\frac{\partial^2 \pi}{\partial G^2} \leq 0$.

Following Fehr and Schmidt (1999), consider individual B's preferences as:

$$U_B = \begin{cases} \pi_B - \alpha(\pi_A - \pi_B), \text{ if } \pi_B \leq \pi_A \\ \pi_B - \beta(\pi_B - \pi_A), \text{ if } \pi_B > \pi_A. \end{cases} \tag{1}$$

For guilt parameter β, Fehr and Schmidt (1999) assume $0 \leq \beta < 1$. This assumption is also made in this study. For the envy parameter α, assume that $0 \leq \alpha$.[1] Previous studies (e.g., Buckley and Croson, 2006; Blanco et al., 2011) study the case when $\frac{\partial \pi}{\partial G} > 0$, and $\frac{\partial^2 \pi}{\partial G^2} = 0$. In this paper, the Cobb-Douglas function of $\pi(x_i, G) = \gamma \log x_i + (1-\gamma)\log G$, where $\gamma \in (0,1)$ is examined for the case when MRS is not constant. Then, individual B's contribution g_B can be found by solving

[1] Fehr and Schmidt (1999) further assume $\beta \leq \alpha$.

$$\max_{x_B, g_B} U_B = \begin{cases} -\alpha\left[\gamma \log\ x_A + (1-\gamma)\log G\right] + (1+\alpha)\left[\gamma \log x_B + (1-\gamma)\log G\right], \\ \qquad \text{if } x_B \leq x_A \\ \beta\left[\gamma \log\ x_A + (1-\gamma)\log G\right] + (1-\beta)\left[\gamma \log x_B + (1-\gamma)\log G\right], \\ \qquad \text{if } x_B > x_A. \end{cases} \tag{2}$$

$$\text{s.t. } x_A + g_A = w,\ x_B + g_B = w,\ g_A + g_B = G.$$

Following Bergstrom et al. (1986), it is assumed that individual B takes the contribution of A as exogenously given (the Nash assumption). Also assume $g_A \in (0, w)$. By substituting $g_B = G - g_A$ into the above and the budget constraints into the utility function, the optimization problem is equivalent to

$$\max_{G} U_B = \begin{cases} -\alpha\gamma \log\ (w - g_A) + (1+\alpha)\gamma \log(w - G + g_A) + (1-\gamma)\log G, \\ \qquad \text{if } 2g_A \leq G \\ \beta\gamma \log\ (w - g_A) + (1-\beta)\gamma \log(w - G + g_A) + (1-\gamma)\log G, \\ \qquad \text{if } 2g_A > G. \end{cases} \tag{3}$$

This utility function is not differentiable. Therefore, to solve this maximization problem, the problem is splitted into two by adding the conditions $G - 2g_A \geq 0$ and $2g_A - G > 0$ as constraints. This makes U_B differentiable within each sub-problem. Then, each sub-problem can be solved by applying the Kuhn-Tucker conditions, and both interior and corner solutions to the original problem can be obtained by comparing the utility levels of the solutions to the two sub-problems. Using this algorithm, individual B's optimal response g_B^* is shown and studied in the next section.

Note that

$$MRS = \frac{\partial U_B / \partial x_B}{\partial U_B / \partial G} = \frac{(1-k)\,\gamma}{1-\gamma} . \frac{G}{w + g_A - G},$$

where $k = -\alpha, \beta$, which means the MRS between the private and public goods is not constant. Further, the MRS when approximating from the left and right to $G = 2g_A$ differ if $-\alpha \neq \beta$.

3. Comparative statics of optimal response

To derive optimal response $g_B^*\left(g_A\right)$, the problem with constraint $G - 2g_A \geq 0$ is examined. This is the case where individual B's sub-utility is relatively low or equal to that of A. The solutions for this sub-problem are

$$g_B = \begin{cases} \frac{1-\gamma}{1+\alpha\gamma} w - \frac{(1+\alpha)\gamma}{1+\alpha\gamma} g_A, & \text{if } 0 < g_A < g_A^\alpha \\ g_A, & \text{if } g_A^\alpha \leq g_A, \end{cases} \tag{4}$$

where $g_A^\alpha = \frac{1-\gamma}{1+\gamma+2\alpha\gamma} w$.[2]

[2] The Kuhn-Tucker conditions for this and the other cases are provided by Yokoo (2018).

Next, examine the problem with the constraint of $2g_A - G > 0$, which is the case where individual B's subutility is relatively high. In this paper, the case when $0 \leq \beta < 1 - \frac{1-\gamma}{\gamma}$ is examined.[3] The solutions for this sub-problem are

$$g_B = \begin{cases} \frac{1-\gamma}{1-\beta\gamma} w - \frac{(1-\beta)\gamma}{1-\beta\gamma} g_A, & \text{if } \underline{g_A^\beta} < g_A \leq \overline{g_A^\beta} \\ 0, & \text{if } \overline{g_A^\beta} \leq g_A < w, \end{cases} \tag{5}$$

where $\underline{g_A^\beta} = \frac{1-\gamma}{1+\gamma-2\beta\gamma} w$ and $\overline{g_A^\beta} = \frac{1-\gamma}{(1-\beta)\gamma} w$.

If $g_B = g_A$, then the level of U_B is independent of α and β. As a result, the solutions of the two sub-problems are compared to derive the optimal response:

$$g_B^* = \begin{cases} \frac{1-\gamma}{1+\alpha\gamma} w - \frac{(1+\alpha)\gamma}{1+\alpha\gamma} g_A, & \text{for } 0 < g_A < g_A^\alpha \\ g_A, & \text{for } g_A^\alpha \leq g_A \leq \underline{g_A^\beta} \\ \frac{1-\gamma}{1-\beta\gamma} w - \frac{(1-\beta)\gamma}{1-\beta\gamma} g_A, & \text{for } \underline{g_A^\beta} < g_A \leq \overline{g_A^\beta} \\ 0, & \text{for } \overline{g_A^\beta} \leq g_A < w. \end{cases} \tag{6}$$

As long as individual B is inequality averse ($0 < \alpha$ and $0 < \beta$), this contribution function is non-monotonic in that

[3] See Yokoo (2018) for the case when $1 - \frac{1-\gamma}{\gamma} \leq \beta < 1$. Note that if $1 - \frac{1-\gamma}{\gamma} \leq \beta$, then $w \leq \overline{g_A^\beta}$.

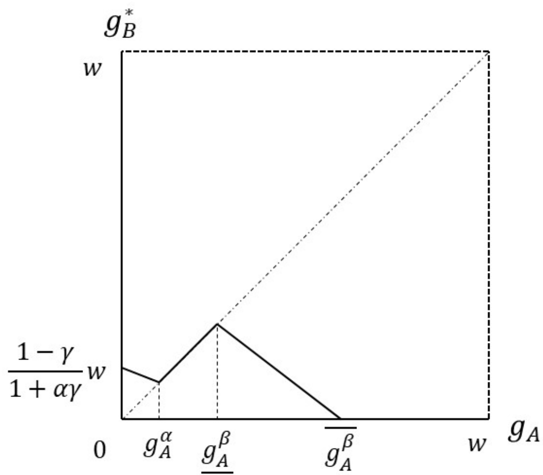

Figure 1: The optimal response $(0 < \beta < 1 - \frac{1-\gamma}{\gamma})$

$$\frac{\partial g_B^*}{\partial g_A} \begin{cases} < 0, & \text{if } 0 < g_A < g_A^\alpha \\ > 0, & \text{if } g_A^\alpha \leq g_A \leq \underline{g_A^\beta} \\ < 0, & \text{if } \underline{g_A^\beta} < g_A \leq \overline{g_A^\beta} \\ = 0, & \text{if } \overline{g_A^\beta} \leq g_A < w. \end{cases} \tag{7}$$

Figure 1 illustrates the contribution function when $0 < \alpha$ and $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$. If individual B believes the other's contribution is low $(0 < g_A < g_A^\alpha)$ or in the range of $\underline{g_A^\beta} < g_A \leq \overline{g_A^\beta}$, B decreases its contribution with the other's increase. If individual B believes the other's contribution is high $(\overline{g_A^\beta} \leq g_A < w)$, B does not contribute at all and free rides. Finally, individual

B increases contribution along with an increase in A's contribution if it is in the range of $g_A^\alpha \leq g_A \leq \underline{g_A^\beta}$, to stay at the kink point of the utility function.

4. Discussion and Conclusions

The literature on the voluntary provision of public goods has studied a general model, including variable MRS (e.g., Warr, 1983; Bergstrom et al., 1986). However, most of the existing models on inequality aversion and public goods study the case with constant MRS. As such, this paper aims to fill this gap in the literature by proposing a model of "kinky" preferences, as in Fehr and Schmidt (1999). An interesting insight from analyzing the comparative statics is that an individual increases contribution with the same amount as the other individual in a two-person economy, matching it conditional on the other's cooperation.

There are several directions for future research. First, while this paper focuses on the comparative statics of individual behavior, studying equilibrium results is required to understand the impact of kinky inequality aversion on the equilibrium provision of public goods when the MRS is not constant. Second, empirical testing of the model is needed. Several studies adopt sequential public good games to test models of social preferences (e.g., Teyssier, 2012). In addition, Uler (2011) provides an experimental design to study public goods provision when the MRS is not constant. Combining these approaches may enable us to test the model. Finally, using this model to explain public goods provision in a field setting where MRS seems not constant is worth studying.

Acknowledgements

I gratefully acknowledge Nguyen Ngoc Mai for her comments and encouragement, as well as the comments from the participants at the 2017 International Workshop on 'Social preferences and environmental and resource economics' at the University of Manchester. This research project was supported by JSPS KAKENHI, grant number 17K18343.

References

[1] Bergstrom, T., Blume, L. and Varian, H., 1986. On the private provision of public goods. Journal of Public Economics, 29(1), pp.25-49.

[2] Blanco, M., Engelmann, D. and Normann, H.T., 2011. A within-subject analysis of other-regarding preferences. Games and Economic Behavior, 72(2), pp.321-338.

[3] Buckley, E. and Croson, R., 2006. Income and wealth heterogeneity in the voluntary provision of linear public goods. Journal of Public Economics, 90(4-5), pp.935-955.

[4] Derin-Güre, P. and Uler, N., 2010. Charitable giving under inequality aversion. Economics Letters, 107(2), pp.208-210.

[5] Engelmann, D., 2012. How not to extend models of inequality aversion. Journal of Economic Behavior & Organization, 81(2), pp.599-605.

[6] Fehr, E. and Schmidt, K.M., 1999. A theory of fairness, competition, and cooperation. Quarterly Journal of Economics, 114(3), pp.817-868.

[7] Teyssier, S., 2012. Inequity and risk aversion in sequential public good games. Public Choice, 151(1-2), pp.91-119.

[8] Uler, N., 2011. Public goods provision, inequality and taxes. Experimental Economics, 14(3), pp.287-306.

[9] Warr, P.G., 1983. The private provision of a public good is independent of the distribution of income. Economics Letters, 13(2-3), pp.207-211.

[10] Yokoo, H.F., 2018. A note on a model of inequality aversion and public goods when the marginal rate of substitution is not constant. Unpublished manuscript. http://www.k-uni.jp/yokoo/research.html