Inequality aversion and the private provision of public goods when the marginal rate of substitution is not constant

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Abstract

This study develops a model of inequality aversion and public goods, which allows the case of the marginal rate of substitution being variable. The utility function of the standard public goods model is nested in the Fehr-Schmidt model. An individual's contribution function for a public good is derived by solving the problem of a utility function that has a kink, and examining both interior and corner solutions. Due to the variable marginal rate of substitution and kinky preferences, the derived contribution function is not monotonic with respect to the other's provision.

Keywords: Inequality Aversion, Kinky Preferences, Private Provision of Public Goods, Variable Marginal Rate of Substitution

JEL Codes: H41, D63, D91

1. Introduction

Several studies have examined the association between inequality aversion and the voluntary provision of public goods (Buckley and Croson, 2006; Blanco et al., 2011; Teyssier, 2012) by testing the models of inequality aversion, including the one developed by Fehr and Schmidt (1999), through laboratory experiments. All of these studies have used a linear public goods game by assuming the marginal rate of substitution (MRS) between the private and public good constant. The original Fehr-Schmidt model sufficiently explains decision making in a

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standard linear public goods game; however, the assumption made by these previous studies (namely, the constant MRS) may be too strong for the demand of public goods outiside the laboratory.

Thus, this study extends the model of inequality aversion and public goods to incorporate the case when the MRS is not constant. To this end, a simple model of a two-person economy is developed to study individuals' contribution to a public good. The theoretical foundation of the model is provided by nesting the utitlity function of the standard public goods model (e.g., Warr, 1983; Bergstrom et al., 1986) into the Fehr-Schmidt model. A maximization problem with a utility function with a kink is presented. To obtain the intuition of the model, a Cobb-Douglas function is adopted, and the problem is solved to derive an optimal response. A clear and empirically testable prediction is derived by the comparative statics.

Engelmann (2012) demonstrates that extending the Fehr-Schmidt model by adding a term for efficiency concerns is misguided, since it is equivalent to a much simpler change. This argument also applies to adding a term for public goods if MRS is constant.[1] Instead of an attempt such as those criticized by Engelmann (2012), this study adds a term that is a concave function of the total amount of the public good provision. As a result, this study proposes a utility function that is non-differentiable by nature and examines both interior and corner solutions. A simple procedure to solve a problem with a non-differentiable function is provided to derive the optimal contribution as a function of the other's contribution. The derived optimal contribution exhibits an inverted N-shape relationship, suggesting that an individual decreases its contribution in response to the other's increase over a specific range, but increases it in the other ranges.

[1] Derin-Güre and Uler's (2010) study is an exception in taking a novel approach to model inequality aversion and public goods. In addition to standard private and public good terms, their utility function includes a term of the concave function of inequality in private good consumption. Their utility function exhibits a variable MRS, which is assumed to be differentiable. Consequently, the study focuses on an interior solution.

2. Model

Consider a model in which there is one private good, one public good and two individuals, A and B. Each individual i consumes an amount x_i of the private good and donates an amount g_i to the supply of the public good. Let $G = g_A + g_B$ be the total private contributions to the public good. Both individuals i are endowed with wealth $w > 0$, which they allocate between private good x_i and contribution g_i. Let $\pi_i = \pi(x_i, G)$ be individual i's sub-utility, which corresponds to monetary payoffs in the Fehr-Schmidt model. Assume that $\frac{\partial \pi}{\partial x} \geq 0$, $\frac{\partial^2 \pi}{\partial x^2} \leq 0$, $\frac{\partial \pi}{\partial G} \geq 0$, and $\frac{\partial^2 \pi}{\partial G^2} \leq 0$.

Following Fehr and Schmidt (1999), consider individual B's preferences as follows:

$$U_B = \pi_B - \alpha \max\{\pi_A - \pi_B, 0\} - \beta \max\{\pi_B - \pi_A, 0\}.$$

For guilt parameter β, Fehr and Schmidt (1999) assume $0 \leq \beta < 1$. This study also adopts this assumption. For the envy parameter α, assume that $0 \leq \alpha$.[2] Previous studies have examined the case when $\frac{\partial \pi}{\partial G} > 0$, and $\frac{\partial^2 \pi}{\partial G^2} = 0$ (e.g., Buckley and Croson, 2006; Blanco et al., 2011). In this study, the Cobb-Douglas function of $\pi(x_i, G) = \gamma \log x_i + (1-\gamma)\log G$, where $\gamma \in (0,1)$ is examined for the case when MRS is not constant. Then, individual B's contribution g_B can be found by solving

$$\max_{x_B, g_B} U_B = \begin{cases} -\alpha\left[\gamma \log\ x_A + (1-\gamma)\log G\right] + (1+\alpha)\left[\gamma \log x_B + (1-\gamma)\log G\right] \\ \qquad \text{if } \pi_B \leq \pi_A, \\ \beta\left[\gamma \log\ x_A + (1-\gamma)\log G\right] + (1-\beta)\left[\gamma \log x_B + (1-\gamma)\log G\right] \\ \qquad \text{if } \pi_B > \pi_A. \end{cases}$$

[2] Fehr and Schmidt (1999) further assume $\beta \leq \alpha$.

$$\text{s.t. } x_A + g_A = w,\ x_B + g_B = w,\ g_A + g_B = G.$$

Following Bergstrom et al. (1986), it is assumed that individual B takes the contribution of A as exogenously given (the Nash assumption). Moreover, the study assumes $g_A \in (0, w)$. By substituting $g_B = G - g_A$ into the above and the budget constraints into the utility function, the optimization problem is equivalent to

$$\max_G U_B = \begin{cases} -\alpha\gamma \log\ (w - g_A) + (1+\alpha)\gamma \log(w - G + g_A) + (1-\gamma)\log G \\ \qquad \text{and } 2g_A \leq G, \\ \beta\gamma \log\ (w - g_A) + (1-\beta)\gamma \log(w - G + g_A) + (1-\gamma)\log G \\ \qquad \text{and } 2g_A > G. \end{cases}$$

This utility function is not differentiable if $\alpha \neq 0$ or $\beta \neq 0$. Therefore, to solve this maximization problem, it is split into two by adding the conditions $G - 2g_A \geq 0$ and $2g_A - G > 0$ as constraints. This makes U_B differentiable within each sub-problem. Then, each sub-problem can be solved by applying the Kuhn-Tucker conditions, and both interior and corner solutions to the original problem can be obtained by comparing the utility levels of the solutions to the two sub-problems. Using this procedure, individual B's optimal response g_B^* is shown and studied in the next section.

Note that

$$MRS = \frac{\partial U_B / \partial x_B}{\partial U_B / \partial G} = \frac{(1-k)\gamma}{1-\gamma} . \frac{G}{w + g_A - G},$$

where $k = -\alpha, \beta$, which means the MRS between the private and public goods is not constant. Further, the MRS when approximating from the left and right to $G = 2g_A$ differ if $-\alpha \neq \beta$.

3. Comparative statics of optimal response

To derive optimal response $g_B^*(g_A)$, the study considers the problem with constraint $2g_A \leq G$. This is the case wherein individual B's sub-utility is relatively low or equal to that of A. By solving this sub-problem, the solution can be written as[3]

$$g_B = \begin{cases} \frac{1-\gamma}{1+\alpha\gamma}w - \frac{(1+\alpha)\gamma}{1+\alpha\gamma}g_A & \text{for } 0 < g_A < g_A^\alpha, \\ g_A & \text{for } g_A^\alpha \leq g_A < w, \end{cases}$$

where $g_A^\alpha = \frac{1-\gamma}{1+\gamma+2\alpha\gamma}w$.

Next, the study considers the problem with the constraint of $g_A \leq G \leq 2g_A$, which is the case wherein individual B's sub-utility is relatively high. This study examines the case when $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$.[4] By solving this sub-problem, the solution can be written as

$$g_B = \begin{cases} g_A & \text{for } 0 < g_A \leq \underline{g_A^\beta}, \\ \frac{1-\gamma}{1-\beta\gamma}w - \frac{(1-\beta)\gamma}{1-\beta\gamma}g_A & \text{for } \underline{g_A^\beta} < g_A < \overline{g_A^\beta}, \\ 0 & \text{for } \overline{g_A^\beta} \leq g_A < w, \end{cases}$$

where $\underline{g_A^\beta} = \frac{1-\gamma}{1+\gamma-2\beta\gamma}w$ and $\overline{g_A^\beta} = \frac{1-\gamma}{(1-\beta)\gamma}w$.

If $g_B = g_A$, then the level of U_B is independent of α and β. Based on this observation, the solutions of the two sub-problems are compared to derive the optimal response of inequality averse ($0 < \alpha$ and $0 < \beta$):

[3] The Kuhn-Tucker conditions for this and the other cases are provided in the Online Appendix.

[4] See the Online Appendix for the case when $1 - \frac{1-\gamma}{\gamma} \leq \beta < 1$. Note that if $1 - \frac{1-\gamma}{\gamma} \leq \beta$, then $w \leq \overline{g_A^\beta}$.

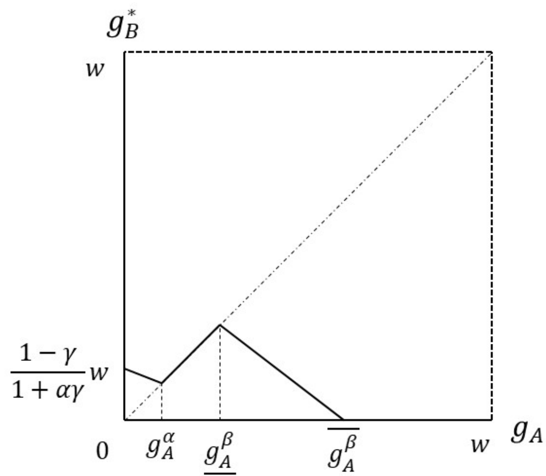

Figure 1: The optimal response ($0 < \alpha$ and $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$)

$$g_B^* = \begin{cases} \frac{1-\gamma}{1+\alpha\gamma}w - \frac{(1+\alpha)\gamma}{1+\alpha\gamma}g_A & \text{for } 0 < g_A < g_A^\alpha, \\ g_A & \text{for } g_A^\alpha \le g_A \le \underline{g_A^\beta}, \\ \frac{1-\gamma}{1-\beta\gamma}w - \frac{(1-\beta)\gamma}{1-\beta\gamma}g_A & \text{for } \underline{g_A^\beta} < g_A < \overline{g_A^\beta}, \\ 0 & \text{for } \overline{g_A^\beta} \le g_A < w. \end{cases}$$

This contribution function is non-monotonic in that

$$\frac{\partial g_B^*}{\partial g_A} \begin{cases} < 0 & \text{for } 0 < g_A < g_A^\alpha, \\ > 0 & \text{for } g_A^\alpha \le g_A \le \underline{g_A^\beta}, \\ < 0 & \text{for } \underline{g_A^\beta} < g_A < \overline{g_A^\beta}, \\ = 0 & \text{for } \overline{g_A^\beta} \le g_A < w. \end{cases}$$

Figure 1 illustrates the contribution function when $0 < \alpha$ and $0 < \beta < 1 - \frac{1-\gamma}{\gamma}$. If individual B believes the other's contribution is low ($0 < g_A < g_A^{\alpha}$) or in the range of $\underline{g_A^{\beta}} < g_A < \overline{g_A^{\beta}}$, B decreases its contribution with the other's increase. If individual B believes the other's contribution is high ($\overline{g_A^{\beta}} \leq g_A < w$), B does not contribute at all and becomes a free rider. Finally, if it is in the range of $g_A^{\alpha} \leq g_A \leq \underline{g_A^{\beta}}$, individual B increases contribution along with an increase in A's contribution to stay at the kink point of the utility function.

Note that when $\alpha = \beta = 0$, $U_B = \pi_B$ and the optimal response is

$$g_B^* = \begin{cases} (1-\gamma)\,w - \gamma g_A & \text{for } 0 < g_A < \frac{1-\gamma}{\gamma} w, \\ 0 & \text{for } \frac{1-\gamma}{\gamma} w \leq g_A < w. \end{cases}$$

This implies that $\frac{\partial g_B^*}{\partial g_A} \leq 0$ for $0 < g_A < w$.

4. Discussion and Conclusions

The literature on the voluntary provision of public goods has presented a general model that includes variable MRS (e.g., Warr, 1983; Bergstrom et al., 1986); most of the existing studies on inequality aversion and public goods have examined cases with constant MRS. To fill this gap in the literature, this study proposes a model of voluntary provision of public goods with variable MRS, and "kinky" inequality aversion, as developed by Fehr and Schmidt (1999). An interesting insight from analyzing the comparative statics is that an individual increases contribution with the same amount as the other individual in a two-person economy, matching it conditional on the other's cooperation.

There are several directions for future research. First, while this study focuses on the comparative statics of individual behavior, a study of the equilibrium results is required to understand the impact of kinky inequality aversion on the equilibrium provision of public goods when the MRS is not constant. Second, empirical testing of the model is needed. Several studies have adopted sequential public good games to test models of social preferences (e.g., Teyssier, 2012). In addition, Uler (2011) provides an experimental design to study public goods provision when the MRS is not constant. Combining these approaches may enable us to test the model. Finally, it is meaningful to explaining public goods provision in a field setting where MRS seems not constant.

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